## Lesson 12. Spatial Poisson Processes

- We have been looking at the arrivals of a Poisson process in a <u>temporal</u> manner

    - e.g. helicopters arriving at a maintenance facility throughout the day

- We can also look at the arrivals of a Poisson process in a <u>spatial</u> manner

    - e.g. locations of emergency response units along a main thoroughfare

- Here, we limit ourselves to one-dimensional spaces (lines), but these ideas can be extended to higher dimensions (2D planes, 3D space)

- We'll talk about why modeling locations along a line as a Poisson process makes sense

Example 1. Police cars are distributed along Main Street in Simplexville according to a Poisson process with rate $\lambda = 1/2$ car per mile:
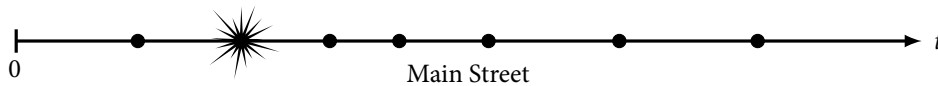

Suppose a crime occurs at the mile 3 marker of Main Street. What is the probability that the nearest police car is within 0.5 miles?

Example 2. In the police car example, what is the probability that there are at least 2 police cars between the mile 11 and mile 14 markers of Main Street? What is the expected number of police cars in that stretch of Main Street?

- So, why does it make sense to model locations along a line as a Poisson process?

- Let's perform an experiment

  ⚠ The code for this experiment is in the accompanying Jupyter notebook for this lesson

- First, we distribute M locations randomly along the interval $[0, L]$ according to the uniform distribution:
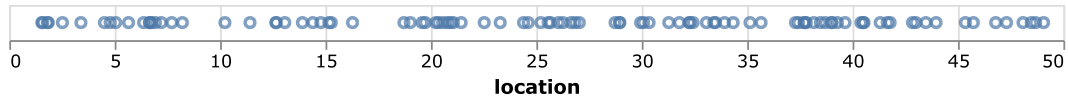
- Next, we compute the *interlocation distances*, or the distances between consecutive points along the interval

- Finally, we create a histogram of these interlocation distances – what does it look like?
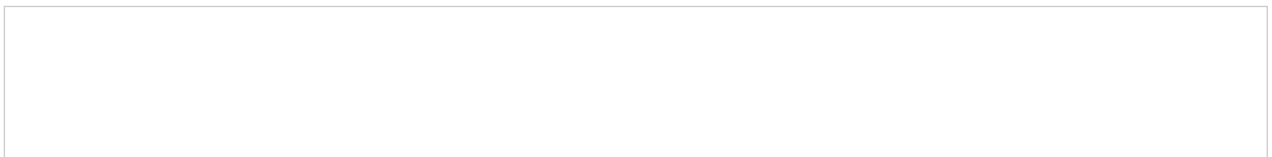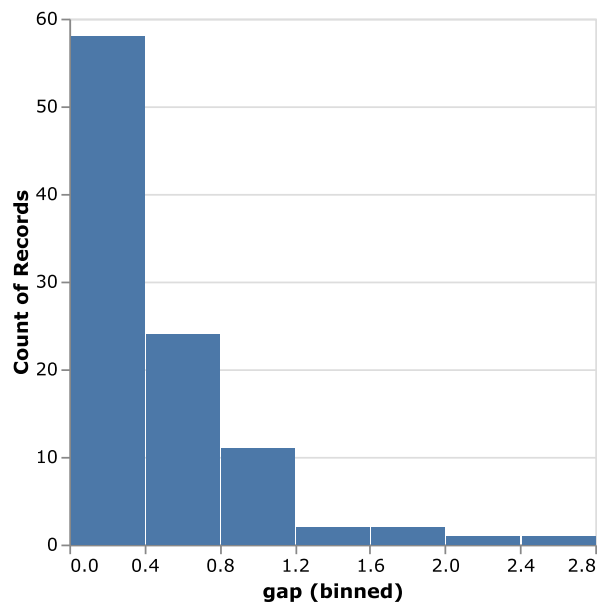
Theorem. Given that M arrivals of a Poisson process have taken place in the interval $[0, L]$, the times at which these arrivals occur can be seen as a M independent samples from Uniform$[0, L]$.

- There are definitely situations in which locations are uniformly distributed throughout a region

⇒ We can use Poisson processes to model and analyze such situations

- This theorem generalizes to higher-dimensional notions of spatial Poisson processes

Example 3. Back to the police car example. Suppose there are 5 police cars between mile 2 and mile 12 of Main Street. What is the probability that 2 of these cars are between mile 6 and mile 12?